The conflict that came with markets. Welfare economics in the history of economic thought

by Niels Peter Hahnemann

Abstract: The equivalence of Walrasian market equilibrium and Pareto optimality has become a benchmark against which actual markets are judged. The ruling has been that there is a need to bring in government to correct error. The present paper examines the theoretical basis for this ruling. The welfare theorems are seen to bring out the impossibilities and “cannot do’s” involved in decision-making as opposed to coercion. Externalities, uncertainty, and information affect the trade in all commodities, not just in particular sorts. This result affects both economic and political social choice. The theorems describe a conflict of principles, which one may claim is an intrinsic characteristic of democracy. Democracy and economics are seen as two sides of the same phenomenon: the market economy.

Keywords: welfare theorems, externality, optimality, equilibrium, own rate of interest, JEL classification system B21, D52, D60, D62, H41.

Introduction

The welfare theorems of modern economics were established in the middle of the twentieth century on the back of classical, neoclassical and macro economics. According to classical economics, economic welfare was regarded as a state of society that could be obtained if the enlightened and self-interested behaviour of the individual corresponded to the common good. Economic welfare referred to a situation where wealth and the options for choice that wealth entails were as large as possible. It would be a state where desires and satisfactions corresponded. David Ricardo had in 1817 described how wealth, or riches, differed from cumulated value. Wealth depended on the quantity of goods for either necessaries or enjoyments, while value depended on the quantity of labour bestowed on their production. Economic rent was an effect of comparative differences in the facility or difficulty of production caused by limited availability of physical resources – land primarily. Rent was a deduction from value that redistributed wealth according to any discrepancy between the outcome of self-interested behaviour and the common

1 A preliminary version of this paper was presented at the biannual conference of the Danish Economic Association, Koldingfjord, January 2012. The author is part-time associate professor, University of Copenhagen and economist, Danmarks Nationalbank.
good. Thus, Ricardo saw rent as a symptom, never a cause of wealth. On the back of this analysis, the conclusions of modern economics came to be that economic welfare would be largest when the rent or quasi-rent discussed by classical, neoclassical and macro economics was as small as possible and redistributed through taxation and public spending. That would correspond to a state where the producer surplus and its deductions from the equivalent consumer surplus was minimised, thus approaching as much as possible so-called perfect competition.

The problem was, of course, how such a state was to be achieved if it could not be done by a system in quantities and prices, that is, by the economy itself. It was not self-evident that desires and satisfactions would correspond if it required a reference to some truth concept reflecting the coincidence of self-interested and common good outcomes in a particular case. It involved the need for judgement. The difficulty in such a ruling became clear through the elaboration of the so-called fundamental welfare theorems of modern economics. These theorems described the welfare of an economy as a situation in which there was equivalence of so-called Walrasian market equilibrium and Pareto optimality:

1) Walrasian market equilibrium outlined the choice of an appropriate set of prices that would eliminate excess demand under conditions of ordinal utility. In ordinal utility choices are ranked according to the preferences of each single individual – not, as in cardinal utility, according to some overall ranking valid for everybody. It was Leon Walras who in 1874 firstly had employed the ordinal approach to utility. 2) Pareto optimality outlined the choice of an appropriate set of prices that would only be altered by making at least one person worse off. Such an alteration would violate the preferences of at least one individual, thereby submitting choice to some overall ranking enforced on everybody.

In modern economics, Walrasian equilibrium represented the self-interested human action, Pareto optimality the common good. The welfare theorems then stated the conditions for the two states to be equivalent. The equivalence of Walra-

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2 See Chapters 2 and 20 in David Ricardo (1817), On the Principles of Political Economy and Taxation.

3 The standard reference here is A. C. Pigou’s monograph from 1912, but the traditional pre-Arrow approach to welfare economics is perhaps best represented by chapter VIII in Paul A. Samuelson (1947), Foundations of Economic Analysis. For an up-to-date textbook discussion, see Allan Feldman, Roberto Serrano (2006), Welfare Economics and Social Choice Theory.
sian market equilibrium and Pareto optimality became a benchmark against which actual markets were judged. The ruling was that markets were incomplete. Only few markets were seen to be able to operate without an external enforcer, and market participants were furthermore anticipated to act irrationally from time to time. If markets thus were regarded as incomplete, and when markets furthermore failed, the need arose to bring in government to correct error. In the present paper I want to examine the theoretical basis for this ruling.

The paper is organised as follows. The next section discusses the first and second fundamental welfare theorems. They are seen to bring out the impossibilities and “cannot do”’s involved in decision-making, which thus is demonstrated to be the opposite of coercion that would follow from imposing social preferences. The following section discusses the third fundamental theorem. The achievement of the common good was here seen to depend on the conditions for individual choice. Any decision has a social dimension, but given the individual orderings a rule that produces a corresponding social ordering is impossible. The decision as to which preferences are relevant is therefore ultimately an ethical value judgement and not a matter of (economic) taste. The subsequent two sections outline the theoretical discussions in the wake of the welfare theorems on the role of externalities, uncertainty, and information in achieving the common good. These problems affect the trade in all commodities, not just of particular sorts. In the final but one section it is discussed how the three theorems, when viewed together, in effect examine the borderline between the economic and the political. Economics and politics are both fields of social interaction heavily encumbered with the impossibility-problem regarding social choice. The last section gives a conclusion: the welfare theorems bring out a conflict of principles, which one may claim is an intrinsic characteristic of democracy. Democracy and economics become two sides of the same phenomenon: the market economy.

First and second fundamental theorems

The fundamental welfare theorems establish a connection between Pareto optimum and Walrasian market equilibrium mainly through the assumption about independent preferences, which was an outgrowth of Adam Smith’s point about the
benefit of capitalism: self-interest. The independent preferences' assumption was taken both to exclude externalities and to produce the common good outcome because not dictated by some outside sovereign power. 1) The concept of externalities originated with Alfred Marshall’s work. While not using this expression, he constructed in 1890 through careful assumptions an increasing demand and a (almost) declining supply curve for a commodity, where the inclinations of the schedules were conditioned on the exclusion of income effects (from money demand) and (as he called it) internal and external economies. 4 2) The equilibrium concept came from the discussion of an exchange economy with coordination in Francis Y. Edgeworth’s Mathematical Physics from 1881. He had described how all agents in the economy exchange bilaterally without any restrictions, and how this led to prices. The exclusion of externalities in this discussion was implicit through Edgeworth's assumption about indifference according to which preferences were independent of institutional or other conditions in the economy. Externality from indifference was a consequence of preferences. Edgeworth thus described a purely self-referential system. 5

On this basis, the first fundamental welfare theorem was stated by Vilfredo Pareto in 1906 6 and mathematically proved by Oscar Lange about forty years later, 7 though the name "first fundamental theorem" originates with Kenneth Arrow. 8 The theorem states the conditions for a market equilibrium to be optimal in the sense of a normatively good or right state. Lange wanted to dispense with interpersonal comparisons of utility and therefore defined the total welfare of a community not as the sum of the utilities of the individuals represented by a scalar, but as a vector $u = (u_1, u_2, \ldots, u_i)$ where $u_i$ is the utility of the $i^{th}$ individual. A maximum of total

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4 Alfred Marshall (1890), Principles of Economics. An Introductory Text. Book 3, Chapter 3 and Mathematical Appendix 2 is on demand. Book 5, Chapter 3 and Appendix H is on supply and on equilibrium conditions.
5 See Karl Vind (1983), Equilibrium with coordination, and Karl Vind (1986), Exchange Equilibrium. 6 “Diremo che i componenti di una collettività godono, in una certa posizione, del massimo di ofelimità, quando è impossibile allontanarsi pochissimo da quella posizione giovando, o nuocendo, a tutti i componenti la collettività; ogni piccolissimo spostamento da quella posizione avendo necessariamente per effetto di giovare a parte dei componenti la collettività e di nuocere ad altri”: Vilfredo Pareto (1906), Manuale di economia politica, p. 253. “Massimo di ofelimità” is ‘Pareto optimality’ for which Pareto provided an extensive geometrical demonstration that culminated in his chart 50 on the subsequent p. 254.
7 Oscar Lange (1942), The Foundations of Welfare Economics.
8 Mark Blaug (2007), The fundamental theorems of modern welfare economics, historically contemplated.
welfare occurred when conditions could not be changed so as to increase the vector \( u \), that is, when it became impossible to increase the utility of any person without decreasing that of others.\(^9\) Utility would, following Gossen\(^10\), be a function of the quantities of the commodities in the individual's possession \( x_n \), where \( x_n \) is the total quantity of the \( n^{th} \) commodity so that there would be equality of marginal rates of substitution between different commodities if each individual maximised utility, as in Gossen's 2\(^{nd} \) law. The problem could then be solved by maximising individual utility \( u_i \), keeping all the side relations, that is, the utility of all the other individuals from all commodities constant.\(^11\) That could be done without prices and in relative quantities only.

The elimination of excess demand by maximising individual utility was thus demonstrated to be also normatively good or right. However, Lange recognised that this solution was basically arbitrary. A second theorem was needed. Maintaining all side relations constant meant that is was necessary to keep the individual's money income constant, and that necessitated a concept of prices. Money income would be equal to the sum of the value of all commodities in his possession: \( M_i = \Sigma p_{n,i}x_{n,i} \), where \( p \) was determined by supply and demand of each commodity. Aggregate income would then also be constant, but its distribution would be indeterminate. It was a condition for the change in utility for rich and for poor but not for the level at which, for example, the rich person's utility was held constant. It basically amounted to an assumption of no income effects and no externalities. \( p_n \) was relative prices. A concept of the absolute price level, or aggregates, was wanting. In effect, Lange's approach went all the way back to Say and to the quantity equation of the mercantilists, once again with the implicit assumption of \( V = 1 \).

Thus, Lange discussed the principles of the second fundamental welfare theorem in the same paper but did not prove it. It was Kenneth J. Arrow who reformulated the second theorem (together with the first) using mathematical set theory,

\(^9\) A vector is larger than another when at least one of its components is greater than the corresponding component of the other vector and none is less.
\(^10\) The, in his time unrecognized, founder of marginal utility theory Herman Heinrich Gossen, who published his theory of utility maximization in 1854.
\(^11\) Relation (3.5) page 217 in Oscar Lange (1942), The Foundations of Welfare Economics. Lange neither referred to Gossen nor Pareto. Pareto also worked with vectors.
and then gave the proof.\textsuperscript{12} The second theorem states the conditions for the existence of a set of prices so that an optimal or normatively good or right state is a market equilibrium. The theorem is a consequence of the deficiencies, or impossibilities and "cannot do"s, of the first theorem and is basically concerned with the problem of initial endowments related to any covering law based on observation, which the first theorem exemplified. Lange came as far as to recognise that if the distribution of income could be determined, a set of prices could be found that eliminated excess demand. It did not suffice to maximise a given utility-vector $u$. Society would have to choose between different vectors, which only could be ordered through some scalar social valuation function of the utility-vector $W(u)$ through Congress and Parliament or, as Lange said, directly.

Either the most preferred $u$ is chosen by giving individuals' utilities a social valuation (weight), and the distribution of income and of commodities is then adjusted accordingly, or $u$ adjusts itself to the social valuation made directly on the distribution of income and of commodities (by some benevolent dictator deciding who gets how much). Individual's utilities then adjust accordingly (they better be happy about it). In any case, prices would adapt so that the marginal utility of income was the same for each individual. But these conditions necessitated social valuation that, being a scalar, would call for aggregation. Lange concluded that if a social valuation function could be established one could determine a condition, in which the good or right state was a market equilibrium given as equality between the marginal social significance of every commodity in the hands of each individual.\textsuperscript{13}

But he did not prove that a social valuation function could exist. Friedrich Hayek then famously replied with the claim that the knowledge needed to obtain such an equilibrium would require a decentralised price system,\textsuperscript{14} while Arrow later proved the impossibility of such a valuation function on the condition that the ordering was not to be either imposed or dictatorial. Let us look more closely at this argument.

Arrow noted in his 1951-paper that the condition for equality of the marginal rates of substitution between different commodities given in Lange's proof of the

\textsuperscript{12} Kenneth J. Arrow (1951), An Extension of the Basic Theorems of Classical Welfare Economics.
\textsuperscript{13} Relation (5.7), page 221 in Oscar Lange (1942), The Foundations of Welfare Economics.
\textsuperscript{14} Friedrich A. Hayek (1945), The Use of Knowledge in Society.
existence of a socially optimum allocation of economic resources (the first fundamental theorem) was not sufficient. The condition in Lange's proof basically amounted to an assumption of independent preferences and utility maximisation. Arrow pointed out that if any component of a commodity bundle, that is, vector apportioned to an individual entered as a variable into the utility functions of more than one individual, that is, if there were externalities (he did not use this expression) the whole analysis put forward by the first theorem would be vitiated.

The second fundamental welfare theorem was supposed to take care of this problem by the choice of an appropriate price system accompanied by public intervention through a social valuation function, but this necessitated in addition a social value judgement as to the distribution of welfare. This judgement came, regarding both theorems, in the form of convexity assumption taken from set theory and mathematical topology. The assumption of a closed and convex set assures the existence of a price vector \((p_1, \ldots, p_n)\) that determines total value in commodity space \(\Sigma p_n x_n\) such that for all commodity vectors \(x\) in the closed convex set \(A\): 
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\Sigma p_n x_n \leq \Sigma p_n x_n^+ \quad \text{where} \quad x_n^+ \quad \text{is a boundary point of} \quad A.
\]
The social commodity bundle must lie in this closed and convex set. It was this assumption that assured that total value had a unique optimal point thus producing the conclusion of Gossen and Lange that marginal utility would be diminishing. But prices became prior to marginal utilities. Individuals were price takers – an important assumption in the theory of perfect, as opposed to oligopolistic or monopolistic, competition.

Arrow then gave a new proof of the first theorem, which followed trivially from \(x^+\) being a unique optimal point, and the first proof of the second theorem, both relying crucially on the theory of convex sets. The first theorem stated, as in Pareto and Lange, that if a set of prices could be found, which equates supply and demand, the resulting situation would be optimal. Market equilibrium was needed as condition for the determination of a price vector. The optimal point was not necessarily a point of bliss (that is, equal to desires) because the optimal point is not the best point the individual would wish for were he unrestrained from concerns about the common good. There were production restraints. Want is, in accordance with Gossen, only satiable at the margin (this is what utility maximisation is all about) and economics would aim at equilibrating desires (want) with satisfactions. The
second fundamental theorem was therefore necessary as a complement to the first fundamental theorem, but the problem was that it was not valid in complete generality. In the second theorem we so-to-say have to find the set of prices that was assumed in the first. The convexity assumption assures this. Any linear combination of points in the social commodity bundle lies in the set. The aggregate would be equal to the sum of its individual components. It amounts to an assumption that interdependencies do not exist, and interdependencies produce externalities in independent decision making. Returns should be nonincreasing. That is why the theorem is less general. Arrow recognised that his analysis was only the first step in an approach to the problems of social welfare. The problem of determining a social value function, in which utility functions are not independent but interdependent, was still outstanding.¹⁵

The reason why the price system can fail follows from the property of the price vector stipulated by the second theorem: the existence of free goods with zero prices was not allowed as that could not lead to a utility maximising price system and would upset the achievement of the common good. If the price system failed it was because the individual had free access to unrestrained consumption of goods produced at some cost, and utility maximisation had then no meaning. The existence of such free goods, therefore, had to be discounted for the second theorem to hold true. However, free goods would certainly exist as they not merely represented a small segment of markets but an interdependency of every good on all markets. Free goods were a denotation for the costless disposal of surplus products consumed at zero prices.¹⁶ They were goods disposed, but not produced, at zero cost so that the price of one good (zero) interfered with the price of other goods, giving an exception to the rule that prices could be selected by the market system. The con-

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¹⁵ “Let it be assumed that for each consumer and each firm there is no divergence between social and private benefits or costs, that is, a given act of consumption or production yields neither satisfaction nor loss to any member of the society other than the consumer or producer in question”: Kenneth J. Arrow (1951), An Extension of the Basic Theorems of Classical Welfare Economics, page 507-8. This is independence of preferences, assumption 2 in the paper. Assumption 3 is about convexity and diminishing marginal utility. Assumption 4 is about convexity and diminishing returns. See also the section on comments on the assumptions, pp. 527-30. The discussion above follows Arrow's exposition of lemma 1 through 5, pp. 512-13 and the ensuing theorems 2 and 5 on the first, theorems 3, 4 and 6 on the second, fundamental welfare theorems respectively.

¹⁶ This is assumption 5 in Kenneth J. Arrow (1951), An Extension of the Basic Theorems of Classical Welfare Economics. Arrow excluded the existence of free goods through property (d) in theorems 3 and 4.
sequence of this then needed to be discounted through the disallowance of free goods. On this exception the first and the second fundamental welfare theorems would hold true and Arrow then finally proved that, if this was the case, the optimal distribution would be efficient, that is, fully resource exhausting and non-trivial. But with the determination of a social value function still outstanding market failure could not – I repeat: not – be addressed from outside of the market system. It was the other way round: markets were the only way of addressing the problem of independent decision making, but markets came with certain conditions. Through clarification of the impossibilities and “cannot do”s involved, decision-making proved to be truly opposed to the coercion that would follow from imposing social preferences.

**The third fundamental theorem**

The great strength of the two welfare theorems was that they replied to the question of how to achieve the common good, posed by the classical economists, by a demonstration that it depended on the conditions for individual choice. This was not clear when Gossen firstly formulated the principle of decreasing marginal utility and utility maximisation. It was a story about the impossibilities and "cannot do"s of decision-making. It took about one hundred years to tell the story! The conditions for achieving the common good turned out to lie in the constraints on individual decision-making. There was no social value function to refer to. But the first and the second fundamental welfare theorems still pointed to the eventuality, or perhaps ambition, of determining and choosing social states from some sovereign and, thus, coercive position – though they could not prove that it was actually possible. The impossibility of this was brought out in full by the third fundamental welfare theorem, the formalisation and proof of which was the crowning achievement of Arrow's endeavours in welfare economics.\(^{17}\)

The third theorem was developed along the lines pursued in the first and second. It explicitly turned economics into a subject about decision-making and its

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\(^{17}\) Kenneth J. Arrow (1950), *A Difficulty in the Concept of Social Welfare*. The discussion of the now three theorems was combined in Chapter IV on the compensation principle, and in Chapter VIII, in Kenneth J. Arrow (1963), *Social Choice and Individual Values*, Second Edition, where the 1st and 2nd welfare theorems were called "the Pareto principle".
conditions. As decision can be about anything (the only condition being that it is thinkable and thus possible), economics came through the third theorem to be regarded as one manner of addressing social problems in competition with other methods. Thus, Arrow commenced the presentation of the third theorem by noting that there were several methods of social choice: political decision (voting), economic decision (markets), dictatorship (single individual), and convention (tradition, religious code). The concept of social choice comprised, in fact, any decision evaluated from the point of its social “dimensionality”, just as the Pareto principle concerned the common good component of self-interested individual decision-making.

The existence of markets implied that people were voting with their purse every day, not only on election day. It was a bit like the old question of whether choosing, for example, apple over peach was good or bad for society – only in an indirect manner: whether choosing the last apple thus leaving other people with no choice would make you go for peach instead. It is important to note here that economics and politics thus were discussed as two rival, or competing, or alternative methods of dealing with the same problem. People could either, in Arrow's formulation, vote or use markets. Both were seen as an approach to social choice problems. The borderline between economics and politics was moveable. The distribution between the two in a social system would be an empirical question.18

The problem for science was whether decisions, where there were many wills of people involved, could claim the same consistency and rationality as methods where there was only one individual. The consistency in the latter case came as a matter of course by means of coercion. If many wills could not claim consistency then social choice would only be formed on the basis of individual, that is, dictatorial decision-making, or on convention. However, each individual could be assumed to have an ordering of alternative social states. Given these individual orderings, could there be a rule that produced a corresponding social ordering, which by

18 In James M. Buchanan (1954), Social Choice, Democracy, and Free Markets, Arrow was criticised for failing to differentiate between the social group as an object for social choice and its individual components. However, the project of Arrow was precisely, in the footsteps of Enlightenment philosophy, to detect the conditions for individual non-coerced self-interested choice in achieving the common good. The reply to Buchanan would be: who are to decide on these distinctive social issues? In Buchanan's approach the political level, or democracy, becomes separated from the economic.
implication would be optimal in as much as it would be consistent and rational? There could not. Arrow showed that the problem was in the aggregation, that is, in the macroscopic issues related to difficulties of interpersonal comparisons. Any aggregation could lead to social choice patterns, which did not create a ranking of the various alternatives. Aggregation therefore did not imply ordering according to consistent preferences, that is, social preferences were not able to both compare alternatives (for all \( x, y \) either \( x \) is preferred or indifferent to \( y \), or \( y \) is preferred or indifferent to \( x \)) and be transitive (for all \( x, y, z \) if \( x \) is preferred or indifferent to \( y \) and \( y \) is preferred or indifferent to \( z \), \( x \) is preferred or indifferent to \( z \)). It was an impossibility theorem equally valid for economic (market) and political (voting) decision-making.

On the back of the assumption that individual preferences were consistent and behaviour rational, Arrow deducted that social choice – if possible – should follow from knowledge because it implied comparability and transitivity of alternative states. Social choice should emanate from the individual. The social states that Arrow then considered as objects for social choice included various types of collective activity in addition to quantities of commodities, labour, and productive resources allocated by individual choice on a market. Each individual would have a definite ordering of conceivable social states by whatever standards deemed relevant, not necessarily egotistical standards but always according to taste, that is, according to individual preferences.

Arrow’s definition of taste tried to capture self-interested behaviour and its consequence in terms of the common good. Behaviour according to dictatorship or convention was barred from achieving this due to its very nature. Social choice should be dependent upon individual taste. If social choice were independent, it would imply a return to Platonic philosophy, Arrow claimed, where the objective social good exists independently of individual desires. The social good should be positively grounded in the good of the individual. This was according to enlightenment philosophy and classical economics. But it was also an exercise in judge-

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20 These conditions basically reflect axioms I and II in Kenneth J. Arrow (1950), A Difficulty in the Concept of Social Welfare restated on p. 13 in Arrow's Social Choice and Individual Values.
ment regarding a matter supposed to be left to rules and deliberations of and by the people themselves. Why would independent social choice have to be disallowed in the market economy?

Arrow here in fact obliged to tradition in the history of economic thought by, so to say, calling in the judges, that is, by referring the case to an external absolutist arbitrator (a referee): the decision, he explained, as to which preferences were relevant was an ethical value judgement and not a matter of economic taste. No sharp line of division between ethical value and economic taste was possible, by which Arrow understood that it was not a matter for science. This ruling allowed Arrow to make his own judgement: if the social good did not have an independent existence it could only be pursued according to some truth concept if irrelevant interdependencies and externalities, which sapped the foundations of independent decision-making from underneath, were discounted. This was firmly in the footsteps of Alfred Marshall’s deductions about the inclinations of demand and supply curves, which were the first to note the necessity to exclude external economies or diseconomies in equilibrium calculation. Thus followed the actually rather contrafactual conditions in Arrow’s presentation of a social ordering based on enlightenment principles: it should be formed from individual orderings, independent of irrelevant alternatives, non-imposed (by tradition), and non-dictatorial.21

The irrelevancy of irrelevant alternatives condition was a purely ethical value judgement that Arrow exercised in an economic analysis supposed to be carried out according to pure economic non-ethical principles. Arrow then proved that a social welfare function satisfying these conditions and yielding a consistent social ordering was impossible. There was divergence between private and social benefits or, as Daniel Bernouilli would have put it in the 18th century, between desires and satisfactions. It was basically this divergence that caused the need for judgement. Not all decision-making could be left to deliberation and rules when confronted with irrelevant externalities and interdependencies.

21 See p. 337 and 338 in Kenneth J. Arrow (1950), A Difficulty in the Concept of Social Welfare. The discussion above follows, in particular, pp. 332-6 in this paper.
The discussion on externalities

But decision-making was still a matter for markets, for the economy, and for economics. It was not really a non-market deficiency of individual decision-making that caused the divergence. It was rather a fault of markets, or in the approach of economics: "the market mechanism does not create a rational social choice." The objective of the economic programme: to relate the, say, positive and normative outcomes of human action so that they correspond, remained unattended, or elusive. The problem was that externalities could not easily be discounted. This was realised in the process of developing the welfare theorems.

It was Francis Bator who in 1958, that is, in the wake of the development of welfare economics explicitly discussed the destructive relationship between external economies and interdependencies on the one hand, which he termed externalities, and the "duality theorem", as he called it, of the first and second fundamental welfare theorems on the other. He then pointed to the consequences of this relationship: market failure. Importantly, Bator noted that the modern formulation of the problem of direct non-price interactions begged the fundamental question regarding its cause. The theoretical assumption necessary to avoid market failure was convexity, while choice in the real world would have to cope with the second best as a consequence of indivisibilities (lumpiness), existence of public goods, or non-appropriability leading to increasing returns to scale and zero-price problems.

Subsequently, R. H. Coase narrowed down in 1960 the problem to transaction costs, which supposedly allowed the internalisation and thus reduction in incidence of externalities or, as he called it, social costs depending upon their magnitude. It was a manner of addressing market failure through the market mechanism, but it rested on assumptions about the economy that were not explained, and the presumption that there would be no externality problem at zero transaction cost was

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22 P. 59 in Kenneth J. Arrow (1963), Social Choice and Individual Values, Second Edition. Towards the end of this book the connection between the market mechanism, the condition of independence of irrelevant alternatives (implicitly: no externalities and interdependencies), and ordinal utility is clearly stated (Chapter VIII, Section IV.1 pp. 109-111).
not substantiated by the welfare theorems, where transaction costs played no role. It still begged the question.

Finally, in 1962 James Buchanan and Craig Stubblebine proposed to address the externality problem through rigorous and precise definition. They noted that previous definitions had been few and unsatisfactory. On their definition, an externality was present when the utility of the $i^{th}$ individual defined as a vector $u_i = u_i(x_1, x_2, \ldots, x_n, z_1, z_2, \ldots, z_n)$ made utility depend not only on activities, that is, on commodity quantities $x_n$ exclusively under his own control but also on the activities $z_n$ controlled by another individual. The condition for efficiency was that the "cross effect" of $z$ on $x$ should be zero so that marginal utility was not affected. If: $\frac{\partial x}{\partial z} \neq 0$, an externality was present. The argument of Buchanan and Stubblebine was implicitly about an asymmetry as the other individual's utility on their definition only was allowed to depend on activities within his control: $u_j = u_j(z_1, z_2, \ldots, z_n)$ otherwise the two persons $i,j$ could trade the externality problem away. They then produced an argument for the existence of optimal states even with externalities present and, thus, not necessitating government intervention.\(^{25}\) The case for this argument, however, rested on a particular definition: the asymmetry assumption. Consumers were not alike. But why should they be different?

Buchanan carried the argument over to the theory of public choice, the first installment of which was published by him and Gordon Tullock in the same year as the externality paper. The principles of the public choice model were based on an assumption of interdependence costs incurred from non-market decision-making of differing consumers, which made the number of individuals a relevant externality. The supply of a "constitution", that is, of public choice (be it with respect to a kindergarten or a kingdom) was an increasing function of decision-making interdependence costs, because they would as costs of coming to an agreement depend positively on the number of individuals needed to make a decision. The demand for public choice was, on the other hand, a decreasing function of external interdependence costs because they would, as the alternative or opportunity costs of having someone like a dictator deciding on behalf of oneself, depend negatively on the number of individuals. Equilibrium then determined the degree to which it was

\(^{25}\) James M. Buchanan, Wm. Craig Stubblebine (1962), Externality, see relation (1) and (4).
rational to have a non-market constitution. Optimality was consistent with externality defined as the number of persons involved in a collective decision.²⁶ It was a way of saying that externalities could and should be dealt with outside of the market economy.

In effect, the assumption of no externalities became a fundamental property of "Bourbakist" general equilibrium economic models. However, Arrow realised that the externality problem was not one of definition but concerned the operative mechanisms of the market economy. Externalities occurred inevitably, he noted, in the uncertainty caused by the allocation of decision-making over time as in, for example, time preference – and when there is uncertainty, information or knowledge becomes a commodity.²⁷ But not all of the information would be tradable under uncertainty. Thus, if there does not exist markets capable of bearing all kinds of risk and if, by consequence, markets cannot fully handle uncertainty due to the elusive character of information as a commodity, externalities arise. Arrow gave in 1969 the first comprehensive discussion of the problem of externalities that pointed this out. He defined, like Buchanan and Stubblebine, an externality as occurring when one person's consumption enters into another person's utility causing market failure and inefficiency. Arrow then argued that externalities are like any other commodities except that there is no market for them, either because there is no force driving a market in the commodity to equilibrium (if there is only one buyer and one seller) or because prices differ between buyer and seller.²⁸

David Starrett took this argument by Arrow one step further in 1972 by showing that if all externalities, on the condition that they were associated with measurable commodities, were included in the economy-wide net output set, that is, if there were a market for all externalities, this would lead to non-convexities. He did this by considering an extended commodity space of vectors $y_{ijk}$, with $y$ the net output of commodity $k$ by the firm $j$ as observed by firm $i$. Firm $i$ is affected by firm $j$, $y_{ijk}$, $i \neq k$ represented the external effect on $i$ from $j$’s production of $k$. There would be traditional markets for $y_{iik}$ and artificial markets in the externalities $y_{ijk}$ ($i \neq j$). Un-

²⁷ Kenneth J. Arrow (1963), Uncertainty and the Welfare Economics of Medical Care.
under the convexity assumption Starrett showed that economic rent would exist in the economy that traded externalities, which made income transfers possible that could correct or compensate for inefficiencies. But rent entailed non-convexity and non-existence of equilibrium. An efficient artificial market scheme was not possible. The conclusion was that the inefficiencies caused by externalities could not be corrected. Externalities were externalities. Consumers, or producers, could not escape their effects.  

**The discussion on information and uncertainty**

Furthermore, externality from uncertainty could be a feature of the trade in all commodities, not just of particular sorts. The possibility followed from the role of resources in the production of goods. Harold Hotelling initially presented the argument in 1931 when he demonstrated that the price of exhaustible resources follows the rate of interest. In a situation of general economic equilibrium the expected return to capital and to resources-in-the-ground such as oil must be equal. Resources are like commodities that produce commodities, that is, like capital goods and may serve as financial investment objects, the price of which depends on general monetary and financial circumstances in the economy. John Maynard Keynes subsequently took this argument further by arguing in 1936 that the price forward on any commodity (not only resources-in-the-ground), that is, the price of the futures contract for a commodity for delivery say a year hence follows the price forward on money. Keynes referred to Piero Sraffa’s theory from 1932 on the own rate of interest of a commodity. Sraffa had argued that in any forward exchange the money rate plus an imputed commodity rate would be equal to the money rate less the expected price increase of the commodity, so that a real rate of interest exists for each commodity. In equilibrium this real rate must be the same  

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29 David A. Starrett (1972), *Fundamental Nonconvexities in the Theory of Externalities*. Starrett’s argument relied on, and referred explicitly to, the Hahn-Banach theorem about the preservation of norm, which allows for linearization. This theorem was also a fundamental but implicit assumption in the proofs of welfare economics given by Arrow. Significantly, thus, Starrett used the same mathematical assumptions to obtain non-convexities in the artificial markets for externalities as the welfare theorems used to obtain convexities in ordinary commodity markets. The possibility that a different mathematical approach could have altered the analysis was not considered.  

30 Harold Hotelling (1931), *The Economics of Exhaustible Resources*.  


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for every commodity. The implication drawn by Sraffa was that, for a given money rate and commodity prices, the price increases of commodities converge towards an equilibrium real rate of interest.32

Keynes then defined the own interest rate on money just like the own interest rate on any commodity, that is, as the percentage surplus over the spot price of a sum of money supplied forward in, for example, a year. The highest of the own rates of interest will, according to Keynes, always be the rate of interest on money due to the special characteristics of money in a monetary and aggregated macro economy. There is thus an upper limit for the own rate of interest given by the money rate. The argument essentially attributed a financial aspect to any commodity or trade because there was added a (real) time dimension to any exchange. Due to time preference the present price of every commodity tends to fall relative to its expected future price, which must be discounted with a rate of interest that increases with maturity, that is, with time. The normal shape of the forward curve for an investment object and, in fact, of the price forward for any commodity is then given by so-called backwardation, that is, as a price that declines forward with increasing maturity in relation to the spot price.

This treatment of time as a commodity represented by the forward curve has basically been carried over into modern economic theory. The equilibrium-upsetting role of externality on time markets has, however, followed suit. Hotelling referred to Frank Ramsey who in 1928 established the conditions for the discounting of savings so that future consumption could equilibrate current savings. Ramsey showed that a condition for this is an infinite time horizon to the agent’s planning known as transversality, according to which the value of infinite but bounded future would be equal to zero so that its influence on today’s price vanishes.33 The commodity of time could then be subject to an optimisation procedure.

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32 Piero Sraffa (1932), Dr. Hayek on Money and Capital. Sraffa’s argument was given for “all sorts of commodities” for which there is a forward market, while Keynes discussed durable commodities (wheat, copper, house, steel-plant-rate of interest).

33 Ramsey, Frank (1928), A mathematical theory of saving. This was an application of the Bolzano-Weierstrass theorem about boundedness, which is one of the four pillars of the “citadel” of mathematical economics, the other three being the axiom of choice, Hahn-Banach (preservation of norm), and the exclusion of the middle (principle of non-contradiction). See chapter 1 in K. Vela Velupillai (2010), Computable Foundations for Economics.
If the past drives the future, as in the transversality condition, there is at any point in time a final date where information is complete and uncertainty non-existing. The degree of elimination of uncertainty at this point would measure the consistency or strength of a given hypothesis. Of course: if uncertainty were to be fully eliminated, anticipations would be equilibrated with actual outcome. The futures price of a commodity at time $t$ maturing at time $T$, basically a contract for forward delivery of some good, could be determined as the spot price at time $t$ discounted with an interest rate with the same maturity adjusted by the convenience of holding the commodity now rather than in future.\textsuperscript{34} There would for any potential trade be available a perfect rate of discount, and time preference would by the approximation to such a rate in an actual trade seek to, so to say, reverse time by trading a determined measure of future for current discounted goods, that is, by substituting a measure of savings for some consumption. The success of this effort would depend on the strength of the conclusion regarding a measure on which a given trade was based.

General equilibrium models all shared the basic property of the welfare theorems: inefficiency was caused by externality, but externality was also a condition that in a contra-factual way served as theoretical justification for efficiency. The need to disregard externality justified the convexity assumption applied to prove the existence of market equilibrium. Theory assumed convexity to exclude externality. Positivist philosophy of science realised the risk of fallacy build into this argument: theorems could affirm the premise (convexity) while denying the consequent (externality). You could claim that an observed consequence (some data) were an exception to a general law. The premise: convexity and the mathematical-economic model that it argues would still be claimed to hold.\textsuperscript{35} To avoid this fallacy, economics (and, in effect, all science) should be seen as a preliminary and applied science mainly: theorems should be subject to falsification and tested on data. This was, in effect, a recognition that there would be a degree of uncertainty present in any economic model. On the basis of this insight, modern economics argued

\textsuperscript{34} See for example the discussion in chapter 2 of Hélyette Geman (2006), Commodity Derivatives. Modelling and Pricing for Agriculturals, Metals and Energy.

\textsuperscript{35} This was the so-called asymmetry thesis argued by Karl Popper in the 1940s and 50s: a logical difference between the inference rules modus ponens and modus tollens.
based on Arrow that uncertainty could be addressed as a commodity of non-tradable information, which would work as externality to a model. Information was an exogenous variable that corroborated a given model, thus determining correspondence between data and a model, between observation and theory. Information and the way it is processed defined the causality structure of the model (its capacity to explain and predict) such as under the rational expectations hypothesis that emerged in economic theory in the 1960s and 70s.

This development in economic theory put the concept of information and the manner in which it is processed in a key position. The development can be regarded as an outgrowth of modern welfare theory. The non-possibility of interpersonal comparison of utility, going from the concept of cardinal to ordinal utility, created a problem of informational privation. The solution to this problem was still nothing more than a ruling. It required an explicit calling in of the judges, the necessity of which had not been fully realised hitherto by economic theory. The decisions of the judges would be based on information but, as an exogenous variable, information would always be in limited supply. Positivism thus gave tenure to Dr Pangloss: things are as they are supposed to be. Normativity is not positive science; welfare economics is ethics and politics; public goods and social choice are externalities.

In sum, the approach to the externality problem became dependent upon the treatment of knowledge and its basis in assumptions about information. The externality problem was analysed, for example, in terms of the principal-agent situation that follows when there is imperfect or asymmetric information. So-called moral hazard and adverse selection are key examples of principal-agent problems caused by informational asymmetries due to the existence of externalities. One of the first instalments in this literature, whose importance has only grown not least since the global financial crisis in 2008, was by James Mirrlees in 1974. He regarded Arrow’s welfare economics as unsatisfactory in a world with imperfect and unreliable information. Crucially, the government could in some respects be regarded as better informed, which gave a basis for a discussion of the so-called second best problem, that is, second to the first best of perfect competition.

The case Mirrlees analysed was characterised by misleading, that is, asymmetric but not imperfect information. He noted that the fundamental welfare theorems
implicitly assumed that consumers revealed the necessary information about themselves without regard to their own self-interest. Thus, information was not treated as a tradable commodity but rather as a pure externality. Information was either costlessly available or totally unusable. The utility function $u(x,h)$ of a household had as arguments both trades $x$ with the rest of the economy and the probability density $h$ of output $y$ given labour input $z$. Utility depended on possible “states of nature” represented by some information set. The optimum of the first and second fundamental welfare theorems was only a competitive equilibrium if there existed a suitable distribution of budgets defined by a function $b(h)$. The variable represented an additional condition, namely that consumers revealed the necessary information about themselves without regard to their own self-interest. But there was no incentive for them to do this, Mirrlees noted.  

Information and the associated state-of-nature variable became the externality that provided a platform for government intervention. Now it was government, not markets that pushed for the optimal, but not competitive, second-best equilibrium. Modern macroeconomic theory and policy have to a large extent built upon these initial attempts at addressing the externality problem. But it does not make the externality problem go away. The welfare theorems concerned the relationship between the efficient outcome and the right or good outcome. With externalities and interdependencies markets were incomplete and market equilibrium inefficient. The welfare theorems thus reflected a growing belief that there was an inherent problem of indeterminacy in markets that led to the assumption of some key role for policies or non-market measures in order to achieve optimality.

**Economics and politics**

To a degree the three theorems can be understood as an investigation of the borderline between the economic and the political explored by the social construct of the individual (the firm or the household). The impossibility that hampered the economy was also an impossibility of political decision-making. There was no "happiness indicator" to impose on the people. Decision-making was free, not coerced. Any existing borderline between economics and politics was thus basically

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36 James A. Mirrlees (1974), Notes on welfare economics, information, and uncertainty.
arbitrary. The theorems called for optimality to be addressed by non-market policy measures, but the theorems at the same time analysed optimality as a market equilibrium problem. Policies were seen as non-market forces that operate through adjustment in one or several exogenous variables of a model. By assumption, an exogenous variable being uncorrelated with the error term of a model, policy could then cause a controllable and targeted change in an endogenous variable.

Policy in effect became a control variable in the achievement of the normatively good or right state – the state, which markets were supposed to achieve. Whether there was correspondence between desires (a model) and satisfactions (the data) became a matter for policies or non-market forces to judge. With correspondence between plan and outcome, desires and satisfactions, the state of the market economy could be ruled as not only optimal but also efficient – but only according to certain conditions constructed outside of markets. The market itself could not decide this, and the welfare theorems reflected the indecision. It caused an analytical indeterminacy that pointed to error-generation and failure of markets if certain conditions did not exist. This would be a twist to the economic programme: externality could be a property of a working market economy – not a cause of market failure. It pointed rather to the existence of incomplete markets. Incompleteness means that there are not enough markets to equalise the balance between utility and risk as when, for example, the contract involved in any trade extends only a limited commitment into the future. Markets would be missing, and the whole project initiated by the classical economists of maximising welfare would be inherently deficient.

However, economics is not only about what people do; it is what they actually do – accordingly they can also do something else. They can, for example, act according to political objectives, or aesthetics, or psychology. People must choose to act according to rational principles, and the decision to do this involves a conflict. Perhaps this was the main message of Arrow’s impossibility theorem, the crowning achievement of modern welfare and general equilibrium economics: to bring attention to the impossibilities and “cannot do”s involved in the decision-making of

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modern society, to note the concerns and the constraints. Amartya Sen then argued in 1970 on the back of Arrow’s result that the impossibility pointed to the need for deontology as opposed to the consequentialism and utilitarianism of modern economic theory. And deontology or, as he called it, proceduralism, necessitated an explicit calling in of the judges.

Sen defined liberty as a minimal condition, according to which choice should be left to the individual himself. Social choice was a discussion of the conditions for a decision being actually left for the individual to make. There should be a nonempty so-called recognised personal sphere for at least two persons (possibly everybody, but if one were enough it would be dictatorship) where each individual’s social choice would be guided entirely by his or her preferences. If \( x \) was preferred to \( y \) by individual \( i \), \( x \) should be judged to be a better state of affairs for society than \( y \). Social choice was the social dimension of any choice, and at least two persons should be capable of making social choice, that is, not being restricted in their individual choice as a condition for minimal liberty. Sen then demonstrated that the combination of this liberty principle, the unanimity implied in the Pareto principle, and a condition that the set of social decisions comes from an unrestricted domain was impossible.\(^{38}\) The line of reasoning of Sen amounted to an explicit calling in of the judges, hitherto being referred to more implicitly by economic theory. He noted that the judge could be an outsider or either of the persons involved making a moral, as implicitly opposed by Sen to economic, judgement.\(^{39}\)

Conclusion

The fundamental welfare theorems detect an important interface between markets, the economy, and democracy. Arrow himself had explicitly noted the difference between the economics of the welfare theorems and the moral and ethical theory dating back to Immanuel Kant. The categorical imperative claimed an objective existence of ethical truths based on rational principles. Arrow regarded Kantian deontology as an idealist concept of a social ordering opposed to the consequentialism of utilitarian economics that justified a pragmatist-only and not mor-

\(^{38}\) Amartya Sen (1970), The Impossibility of a Paretian Liberal.
\(^{39}\) Amartya Sen (1983), Liberty and Social Choice.
al imperative. Sen’s argument was that there was an inconsistency, or conflict, between the Pareto principle and individual liberty that nevertheless justified the deontological principle. In effect, the judges requested by both Arrow and Sen were power and politics — the political class or the politicians.

It is important to understand that this request was based on the three welfare theorems taken together. They argued about the possibility of doing the right thing as a purely economic problem spelled out in prices and quantities and approached through independent supply and demand mappings. The problem of finding a solution to the pragmatist imperative was basically left in the open by the 1st and the 2nd theorems, and the 3rd theorem implied that it could not be solved by political means outside of the market economy. Options such as dictatorship or convention obviously could not represent the common good either. This unsettled problem was not a logical fallacy; it concerned the whole of the really existing economy, and all of it. It was an argument in favour of individual decision-making and about the conditions for this as opposed to coercion. But the conditions needed interpretation by the judges. It is the impossibility of these judges’ job in the context of the economic quest for government based on the welfare theorems that brings out a conflict of principles, which one may claim is an intrinsic characteristic of democracy. This conflict was not immediately apparent in Arrow’s discussion of the welfare theorems, but its interpretation as an expression of democracy is justified by the entirely political implications of welfare economics.

Based on the welfare theorems welfare economics described the science of government and the art of doing the right thing as the outcome of self-interested and enlightened behaviour. Decision-making was carried out according to a truth concept, but it became increasingly clear through this episode in the course of the history of economic thought that rational choice could not escape an inherent never-ending conflict. While economics, as the ambition of government described by welfare theory, had an object, democracy had no object or issue of its own. Democracy was the conflict that came with markets. Democracy and economics became two sides of the same phenomenon: the market economy.

References


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